J. Sargeant Reynolds Community College
Course Content Summary

Course Prefix and Number: MTH 266 Credits: 3

Course Title: Linear Algebra

Course Description

Covers matrices, vector spaces, determinants, solutions of systems of linear equations, basis and dimension, eigenvalues, and eigenvectors. Features instruction for mathematical, physical and engineering science programs. Prerequisite: Completion of MTH 263 or equivalent with a grade of B or better, or MTH 264 or equivalent with a grade of C or better. Lecture 3 hours. Total 3 hours per week. 3 credits

General Course Purpose

The general purpose is to give the student a solid grasp of the methods and applications of linear algebra, and to prepare the student for further coursework in mathematics, engineering, computer science and the sciences.

Course Prerequisites/Corequisites

Prerequisite: Completion of MTH 263 or equivalent with a grade of B or better or MTH 264 or equivalent with a grade of C or better.

Course Objectives

Upon completing the course, the student will be able to:

Matrices and Systems of Equations

- Use correct matrix terminology to describes various types and features of matrices (triangular, symmetric, row echelon form, et.al.)
- Use Gauss-Jordan elimination to transform a matrix into reduced row echelon form
- Determine conditions such that a given system of equations will have no solution, exactly one solution, or infinitely many solutions
- Write the solution set for a system of linear equations by interpreting the reduced row echelon form of the augmented matrix, including expressing infinitely many solutions in terms of free parameters
- Write and solve a system of equations modeling real world situations such as electric circuits or traffic flow

Matrix Operations and Matrix Inverses

- Perform the operations of matrix-matrix addition, scalar-matrix multiplication, and matrix-matrix multiplication on real and complex valued matrices
- State and prove the algebraic properties of matrix operations
- Find the transpose of a real valued matrix and the conjugate transpose of a complex valued matrix
- Identify if a matrix is symmetric (real valued)
- Find the inverse of a matrix, if it exists, and know conditions for invertibility.
- Use inverses to solve a linear system of equations

Determinants

- Compute the determinant of a square matrix using cofactor expansion
- State, prove, and apply determinant properties, including determinant of a product, inverse, transpose, and diagonal matrix
- Use the determinant to determine whether a matrix is singular or nonsingular

- Use the determinant of a coefficient matrix to determine whether a system of equations has a unique solution

Norm, Inner Product, and Vector Spaces

- Perform operations (addition, scalar multiplication, dot product) on vectors in R^n and interpret in terms of the underlying geometry
- Determine whether a given set with defined operations is a vector space

Basis, Dimension, and Subspaces

- Determine whether a vector is a linear combination of a given set; express a vector as a linear combination of a given set of vectors
- Determine whether a set of vectors is linearly dependent or independent
- Determine bases for and dimension of vector spaces/subspaces and give the dimension of the space
- Prove or disprove that a given subset is a subspace of R^n
- Reduce a spanning set of vectors to a basis
- Extend a linearly independent set of vectors to a basis
- Find a basis for the column space or row space and the rank of a matrix
- Make determinations concerning independence, spanning, basis, dimension, orthogonality and orthonormality with regards to vector spaces

Linear Transformations

- Use matrix transformations to perform rotations, reflections, and dilations in R^n
- Verify whether a transformation is linear
- Perform operations on linear transformations including sum, difference and composition
- Identify whether a linear transformation is one-to-one and/or onto and whether it has an inverse
- Find the matrix corresponding to a given linear transformation T: R^n -> R^m
- Find the kernel and range of a linear transformation
- State and apply the rank-nullity theorem
- Compute the change of basis matrix needed to express a given vector as the coordinate vector with respect to a given basis

Eigenvalues and Eigenvectors

- Calculate the eigenvalues of a square matrix, including complex eigenvalues.
- Calculate the eigenvectors that correspond to a given eigenvalue, including complex eigenvalues and eigenvectors.
- Compute singular values
- Determine if a matrix is diagonalizable
- Diagonalize a matrix

Major Topics to be Included

- Matrices and Systems of Equations
- Matrix Operations and Matrix Inverses
- Determinants
- Norm, Inner Product, and Vector Spaces
- Basis, Dimension, and Subspaces
- Linear Transformations
- Eigenvalues and Eigenvectors

Effective Date/Updated: August 1, 2022